THE COMPONENTS OF ENERGY RELEASE RATE FOR INTERFACIAL CRACKS

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(Received March 22, 1991)

Interfacial crack growth is inherently mixed mode in nature and mode-mixity must be defined clearly in order to characterize it. Mode Ⅰ and mode Ⅱ strain energy release rates for an interfacial crack in bimaterial system were analytically derived by the virtual crack closure technique. It is shown that the energy release rate for mode Ⅰ and mode Ⅱ do not converge due to the presence of violent oscillatory near tip behavior. However, the total energy release rate is well-defined.

Key Words : Bimaterial System, Interfacial Crack, Energy Release Rate, Virtual Crack Closure Technique, Mode-Mixity

1. INTRODUCTION

The study of crack lying along the interface of two dissimilar elastic media is very important to many of today's advanced materials such as composite materials, adhesively bonded joints and microelectronic devices. In general, interfacial fracture in bimaterial system is different from the homogeneous case. The difference between material properties across the interface disrupt the symmetry even when the geometry and loading are symmetric. Thus, if a crack is constrained to grow along the interface, then the growth is inherently mixed mode in nature and definition of mode Ⅰ and mode Ⅱ fracture toughness needs clarification. For practical purposes, fracture parameters over the full range of mode-mixes must be found in order to characterize the interfacial crack growth completely.

Williams(1959) was the first to investigate this problem and found that the stresses possess an oscillatory character of the type $r^{-1/2}\sin$ (or cos) of the argument $\varepsilon \ln r$ where ε is a bimaterial constant and r is the radial distance from the crack tip. The analysis of interfacial fracture in bimaterial bodies for simple geometries and loadings has been conducted by Erdogan(1963, 1965), Rice and Sih(1965) and England (1965). More complex interfacial crack geometries have been analyzed by numerical methods and some consideration has been given for extracting mode-mixes. A reciprocal work contour integral method for calculating stress intensity factors was extended to interfacial cracks by Hong and Stern (1978). Smelser(1979) suggested an alternate method for obtaining stress intensity factors for bimaterial bodies using numerical crack flank displacement data. Yau and Wang (1982, 1984) formulated an approach based on a conservation laws in elasticity for inhomogeneous solids. Stress intensity factor solutions for each individual fracture mode could be determined accurately and conveniently from information extracted from known auxiliary solutions and conservation integrals along a selected remote path. A technique for extracting mixed-mode interfacial fracture parameters was derived by Sun and Jih(1987), Raju et al.(1988), Hamoush and Ahmad(1989) and Sun and Manoharan(1989) independently. Matos et al.(1989) developed a numerical method for obtaining the fracture parameters based on an evaluation of the J-integral by the virtual crack extension method.

In the present study, mode Ⅰ and mode Ⅱ strain energy release rates for an interfacial crack in bimaterial system were analytically derived by the virtual crack closure technique. Results for a case study were presented.

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2. FORMULATION

The strain energy release rate can be evaluated by the virtual crack closure technique as proposed by Irwin(1957). When the crack extends by an amount δa, the energy release rate in the process is equal to the work required to close the crack to its original length. This can be expressed mathematically as follows ;

$$G=\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}[\sigma_{22}(r)\Delta u_2(\delta a-r) + \sigma_{12}(r)\Delta u_1(\delta a-r)]dr \qquad (1)$$

where $\sigma_{12}(r)$ and $\sigma_{22}(r)$ are the stress components when the crack tip is at $x_1=0$, $\Delta u_1(\delta a-r)$ and $\Delta u_2(\delta a-r)$ are the relative displacements of the crack faces when the crack tip is at $x_1=\delta a$, and δa is the virtual crack extension (Fig. 1). Following the definition of fracture modes in homogeneous materials, the mode Ⅰ and mode Ⅱ components of the energy release rate are

$$G_1=\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}[\sigma_{22}(r)\Delta u_2(\delta a-r)]dr \qquad (2)$$

$$G_2=\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}[\sigma_{12}(r)\Delta u_1(\delta a-r)]dr \qquad (3)$$

Near-tip stress components along $\theta=0$ can be written as

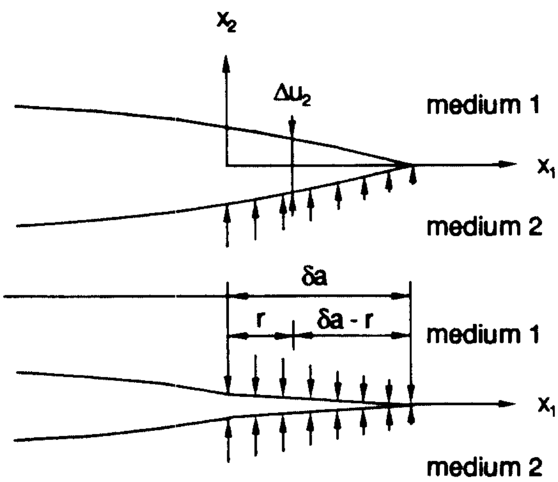

Fig. 1 Closure of crack tip

$$\sigma_{22}(r)=\frac{1}{\sqrt{2\pi r}}[K_1\cos(\varepsilon\ln r)-K_2\sin(\varepsilon\ln r)] \tag{4}$$

$$\sigma_{12}(r)=\frac{1}{\sqrt{2\pi r}}[K_1\sin(\varepsilon\ln r)+K_2\cos(\varepsilon\ln r)] \tag{5}$$

where K is the interfacial stress intensity factor and the bimaterial constant is

$$\varepsilon=\frac{1}{2\pi}\ln\left[\frac{\chi_1\mu_2+\mu_1}{\chi_2\mu_1+\mu_2}\right] \tag{6}$$

where μ is the shear modulus and χ is defined as 3-4ν in plane strain and $(3\text{-}\nu)/(1-\nu)$ in plane stress. The subscripts 1 and 2 refer to the upper and lower materials respectively. The crack face displacements for plane strain at a distance r behind the crack tip are

$$\Delta u_2+i\Delta u_1=\frac{2[(1-\nu_1)/\mu_1+(1-\nu_2)/\mu_2]}{(1+2i\varepsilon)\cosh\pi\varepsilon}K\sqrt{\frac{r}{2\pi}}r^{i\varepsilon} \tag{7}$$

Rewriting the displacement jump components gives

$$\Delta u_2(\delta a-r)=\frac{2[(1-\nu_1)/\mu_1+(1-\nu_2)/\mu_2]}{(1+4\varepsilon^2)\cosh\pi\varepsilon}\sqrt{\frac{\delta a-r}{2\pi}} \tag{8}$$
$$\times\{K_1[\cos\{\varepsilon\ln(\delta a-r)\}+2\varepsilon\sin\{\varepsilon\ln(\delta a-r)\}]$$
$$+K_2[-\sin\{\varepsilon\ln(\delta a-r)\}+2\varepsilon\cos\{\varepsilon\ln(\delta a-r)\}]\}$$

$$\Delta u_1(\delta a-r)=\frac{2[(1-\nu_1)/\mu_1+(1-\nu_2)/\mu_2]}{(1+4\varepsilon^2)\cosh\pi\varepsilon}\sqrt{\frac{\delta a-r}{2\pi}}$$
$$\times\{K_1[\sin\{\varepsilon\ln(\delta a-r)\}-2\varepsilon\cos\{\varepsilon\ln(\delta a-r)\}] \tag{9}$$
$$+\{K_2[\cos\{\varepsilon\ln(\delta a-r)\}+2\varepsilon\sin\{\varepsilon\ln(\delta a-r)\}]\}$$

Substituting the stress and displacement jumps from Eqs.(4) and (8) gives

$$G_1=\frac{[(1-\nu_1)/\mu_1+(1-\nu_2)/\mu_2]}{\pi(1+4\varepsilon^2)\cosh\pi\varepsilon}(I_1K_1^2+I_2K_2^2+I_3K_1K_2) \tag{10}$$

where

$$I_1=\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}\sqrt{\frac{\delta a-r}{r}}[\cos(\varepsilon\ln r)\cos\{\varepsilon\ln(\delta a-r)\}$$
$$+2\varepsilon\cos(\varepsilon\ln r)\sin\{\varepsilon\ln(\delta a-r)\}]dr \tag{11}$$

$$I_2=\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}\sqrt{\frac{\delta a-r}{r}}[\sin(\varepsilon\ln r)\sin\{\varepsilon\ln(\delta a-r)\}$$
$$-2\varepsilon\sin(\varepsilon\ln r)\cos\{\varepsilon\ln(\delta a-r)\}]dr \tag{12}$$

$$I_3=\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}\sqrt{\frac{\delta a-r}{r}}[-\sin\{\varepsilon\ln r\}(\delta a-r)\}$$
$$+2\varepsilon\cos(\varepsilon\ln r(\delta a-r)\}]dr \tag{13}$$

The mode II component can be obtained similarly as

$$G_2=\frac{[(1-\nu_1)/\mu_1+(1+\nu_2)/\mu_2]}{\pi(1+4\varepsilon^2)\cosh\pi z}(I_2K_1^2+I_1K_2^2-I_3K_1K_2) \tag{14}$$

The total energy release rate is the sum of G_1 in Eq.(10) and G_2 in Eq. (14) as follows

$$G_T=G_1+G_2$$
$$=\frac{[(1-\nu_1)/\mu_1+(1-\nu_2)/\mu_2]}{\pi(1+4\varepsilon^2)\cosh\pi\varepsilon}(I_1+I_2)(K_1^2+K_2^2)$$
$$=\frac{[(1-\nu_1)/\mu_1+(1-\nu_2)/\mu_2]}{\pi(1+4\varepsilon^2)\cosh\pi\varepsilon}I_4(K_1^2+K_2^2) \tag{15}$$

where

$$I_4\lim_{\delta a\to o}\frac{1}{2\delta a}\int_0^{\delta a}\sqrt{\frac{\delta a-r}{r}}[\cos(\varepsilon\ln\frac{\delta a-r}{r})$$
$$+2\varepsilon\sin(\varepsilon\ln\frac{\delta a-r}{r})]dr \tag{16}$$

The integral I_4 in (16) can be evaluated using the following changes

$$\int_0^{\delta a}\sqrt{\frac{\delta a-r}{r}}[\cos(\varepsilon\ln\frac{\delta a-r}{r})]dr=\int_0^{\delta a}Re\{\left(\frac{\delta a-r}{r}\right)^{0.5+i\varepsilon}\}dr$$
$$\int_0^{\delta a}\sqrt{\frac{\delta a-r}{r}}[\sin(\varepsilon\ln\frac{\delta a-r}{r})]dr=\int_0^{\delta a}Im\{\left(\frac{\delta a-r}{r}\right)^{0.5+i\varepsilon}\}dr \tag{17}$$

and the relation

$$\int_0^1\left(\frac{1-t}{t}\right)^{0.5+i\varepsilon}dt=\frac{\pi}{2\cosh\pi\varepsilon}(1+2i\varepsilon) \tag{18}$$

If we let $r=t\delta a$ in (17) in order to use the relation (18), then the resulting total energy release rate can be reduced as follows

$$G_T=\frac{1}{4\cosh^2\pi\varepsilon}\left[\frac{1-\nu_1}{\mu_1}+\frac{1-\nu_2}{\mu_2}\right](K_1^2+K_2^2) \tag{19}$$

which coincides with the result by Malyshev and Salganik (1965).

3. RESULTS

Equations(10) and (14) show that the mode I and mode II components of energy release rates are dependent on the choice of δa. As δa approaches zero, sin (or cos) of the argument $\varepsilon\ln(\delta a-r)$ oscillates between +1 and −1. In the finite element analysis, this means that individual components of energy release rates depend on the crack tip element size and do not show the convergence as the crack tip elements are made smaller. In the limit as ε approaches zero, i. e. the case of homogeneous crack, Eqs.(10) and (14) do not depend on δa, and thus can be quickly shown to converge to the plane strain mode I and mode II energy release rates for homogeneous cracks. Eq. (19) shows that the total energy release rate is therefore independent of the choice of δa. This means that the total energy release rate will converge as the size of near tip element decreases although the individual components oscillate.

The example considered was a finite center interfacial crack between two dissimilar media in an infinite plate subjected to a normal stresses at the boundaries as shown in Fig. 2. A problem of the identical loading conditions was solved by Rice and Sih(1965) as follows.

$$K_1=\sqrt{\pi a}\,\sigma_y[\cos(\varepsilon \ln 2a)+2\varepsilon \sin(\varepsilon \ln 2a)] \qquad (20)$$
$$K_2=\sqrt{\pi a}\,\sigma_y[\sin(\varepsilon \ln 2a)-2\varepsilon \cos(\varepsilon \ln 2a)]$$

Using the K_1 and K_2 values in Eq. (20), G_1, G_2 and G_T according to Eqs. (10), (14) and (19), respectively, could be calculated. Simple numerical integration schemes were used to evaluate the finite integrations in Eqs.(10) and (14). Without limiting processes, the results are presented in Figs. 3 and 4.

In Fig. 3, energy release rates are plotted up to $\delta a=0.1$. When δa decreases less than 0.01, the values of G_1 and G_2 show the sharp decrease or increase which could be explained from the oscillatory characteristics. The total energy release rate is constant through whole range of δa. It can be seen that there is a region of δa in which G_1 and G_2 are almost constant. Sun and Jih(1987) proposed that these constant values might be used as the components of energy release rate for practical purposes. To check their suggestions, energy release rates are plotted from $\delta a=1\times10^{-20}$ to 1×10^{5} in Fig. 4. The results show that the values of G_1 and G_2 oscillate in entire region of δa, which means the suggestion is inappropriate.

Interfacial crack growth is inherently mixed mode in nature and mode-mixity must be defined clearly in order to characterize it. The identification of mode-mixity can be based on the stress intensity factors K_1 and K_2 or the energy release rates G_1 and G_2 associated with opening and shear modes of fracture. Due to the lack of convergence of G_1 and G_2, the definition of mode-mixity in terms of the components

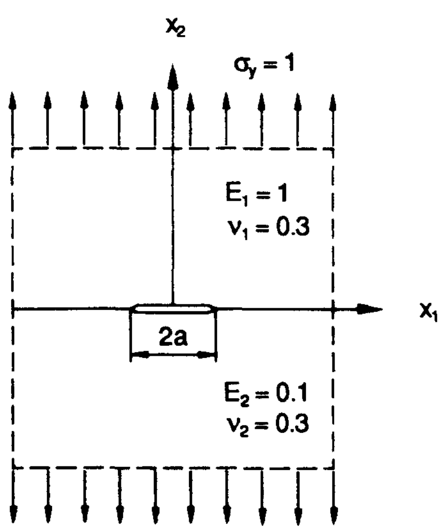

Fig. 2 Infinite plate with center crack under tension($\varepsilon=-0.0758$)

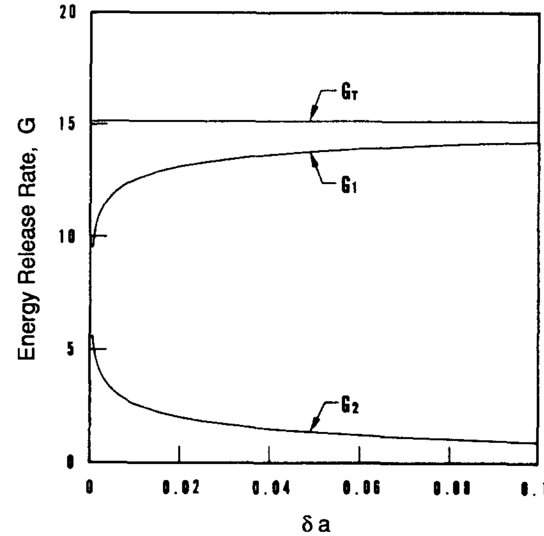

Fig. 3 The components of energy release rate as functions of δa

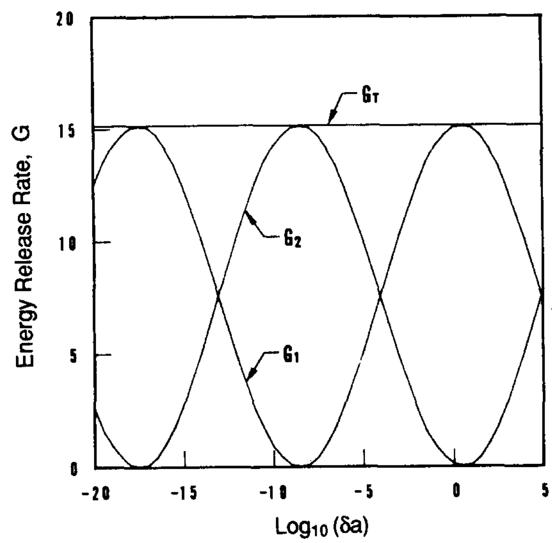

Fig. 4 Dependence of energy release rates on $\log_{10}(\delta a)$

of energy release rate may not be adequate in interfacial cracks. Unless new definitions of G_1 and G_2 are adopted for the interfacial crack, stress intensity factors can be used to define mode-mixity.

4. CONCLUSIONS

By the virtual crack closure technique, it has been derived and shown analytically that the mode I and mode II components of strain energy release rates for the interfacial crack in bimaterial system are not well defined due to the presence of oscillatory terms. However, the total energy release rate does converge. Unless new definitions of G_1 and G_2 are adopted for the interfacial crack, stress intensity factors can be used to define mode-mixity.

REFERENCES

England, A. H., 1965, "A Crack between Dissimilar Media," ASME J. Appl. Mech., Vol. 32, pp. 400～402.

Erdogan, F., 1963, "Stress Distribution in a Non-homogeneous Elastic Plane with Cracks," ASME J. Appl. Mech., Vol. 30, pp. 232～236.

Erdogan, F., 1965, "Stress Distribution in Bonded Dissimilar Materials with Cracks," ASME J. Appl. Mech., Vol 32, pp 403～410.

Hamoush, S. A. and Ahmad, S.H., 1989, "Mode Ⅰ and Mode Ⅱ Stress Intensity Factors for Interfacial Cracks in Bi-Material Media," Eng. Fract. Mech., Vol. 33, No. 3, pp. 421～427.

Hong, C. C. and Stern, M., 1978, "The Computation of Stress Intensity Factors in Dissimilar Materials," J. Elasticity, Vol. 8, pp. 21～34.

Irwin, G.R., 1957, "Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate," J. Appl. Mech., Vol. 24, pp. 361～364.

Malyshev, B. M. and Salganik, R. L., 1965, "The Strength of Adhesive Joints Using Theory of Cracks," Int. J. Fract. Mech., Vol. 1, pp. 114～128.

Matos, P.P.L., McMeeking, R. M., Charalambides, P. G. and Drory, M. D., 1989, "A Method for Calculating Stress Intensities in Bimaterial Fracture," Int. J. Fract., Vol. 40, pp. 235～254.

Raju, I.S., Crews Jr, J. H. and Aminpour, M. A., 1988, "Convergence of Strain Energy Release Rate Components for Edge-Delaminated Composite Laminates," Eng. Fract. Mech., Vol.30, No. 3, pp. 383～396.

Rice, J.R. and Sih, G. C., 1965, "Plane Problems of Cracks in Dissimilar Media," ASME J. Appl. Mech., Vol.32, pp. 418～423.

Smelser, R.E., 1979, "Evaluation of Stress Intensity Factors for Bimaterial Bodies using Numerical Crack Flank Displacement Data," Int. J. Fract., Vol 15, pp. 135～143.

Sun, C. T. and Jih, C. J., 1987, "On Strain Energy Release Rates for Interfacial Cracks in Bi-Material Media," Eng. Fract. Mech., Vol 28, No. 1, pp. 13～20.

Sun, C. T. and Manoharan, M. G., 1989, "Strain Energy Release Rates of an Interfacial Crack between Two Orthotropic Solids," J. Comp. Matls., Vol.23, pp. 461～478.

Wang, S.S. and Yau, J.F., 1982, "Interface Cracks in Adhesively Bonded Lap-Shear Joints," Int. J. Fract., Vol.19, pp. 295～309.

Williams, M. L., 1959, "The Stresses around a Fault or Crack in Dissimilar Media," Bulletin of the Seismological Society of America, Vol.49, pp. 199～204.

Yau, J.F. and Wang S. S., 1984, "An Analysis of Interface Cracks between Dissimilar Isotropic Materials using Conservation Integrals in Elasticity," Eng. Fract. Mech., Vol.20, pp. 423～432.